Course: US01CPHY01

UNIT – 1 ELASTICITY – I

❖ Introduction:

If the distance between any two points in a body remains invariable, the body is said to be a rigid body. In practice it is not possible to have a perfectly rigid body. The deformations are

(i) There may be change in length

(ii) There is a change of volume but no change in shape

(iii) There is a change in shape with no change in volume

All bodies get deformed under the action of force. The size and shape of the body will change on application of force. There is a tendency of body to recover its original size and shape on removal of this force.

Elasticity: The property of a material body to regain its original condition on the removal of deforming forces, is called elasticity. Quartz fibre is considered to be the perfectly elastic body.

Plasticity: The bodies which do not show any tendency to recover their original condition on the removal of deforming forces are called plasticity. Putty is considered to be the perfectly plastic body.

Load: The load is the combination of external forces acting on a body and its effect is to change the form or the dimensions of the body. Any kind of deforming force is known as Load.

When a body is subjected to a force or a system of forces it undergoes a change in size or shape or both. Elastic bodies offer appreciable resistance to the deforming forces. As a result, work has to be done to deform them. This amount of work is stored in body as elastic potential energy. When the deforming force is removed, its increased elastic potential energy produced a tendency in the body to restore the body to its original state of zero energy or stable equilibrium. This tendency is due to the internal forces which come into play by the deformation.

Stress: When a force is applied on a body, there will be relative displacement of the particles. Due to the property of elasticity the particles tend to regain their original position. *The restoring or recovering force per unit area set up inside the body is called stress.*

- The stress is measured in terms of the load or the force applied per unit area. Hence its units are *dynes/cm*^{2} in CGS and *Newton/m*^{2} in MKS.
- It has a dimension **$[ML^{-1}T^{-2}]$.** It is same as that of pressure.

There are two types of stress.

(1) Normal Stress: Restoring force per unit area *perpendicular to the surface* is called normal stress.

(2) Tangential or Shearing Stress: Restoring force per unit area *parallel to the surface* is called tangential or shearing stress.

Strain: The unit change produced in the dimensions of a body under a system of forces in equilibrium, is called strain. The strain being ratio. It has no unit.

There are following three types of strains.

(1) Longitudinal or Linear Strain: It is defined as the increase in length per unit original length of an object when the it is deformed by an external force.

The ratio of change in length to the original length is called longitudinal or elongation strain.

$$i.e.\ \ Longitudinal\ or\ Linear\ Strain = \frac{Change\ in\ length\ (l)}{Original\ length\ (L)}$$

It is also called *Elongation strain* or *Tensile strain*.

(2) Volume Strain: It is defined as change in volume per unit original volume, when an object is deformed by the external force.

The ratio of change in volume to the original volume is called volume strain.

$$i.e.\ \ Volume\ Strain = \frac{Change\ in\ volume\ (v)}{Original\ volume\ (V)}$$

(3) Shear strain: When the force applied is acting parallel to the surface of the body then the change takes place only in the shape of the body. The corresponding strain is called shear strain.

The angular deformation produced by an external force is called shear strain.

Characteristics of a Perfectly Elastic Material

If a body is perfectly elastic then

- Strain is always same for a given stress.
- Strain vanishes completely when the deforming force is removed.
- For maintaining the strain, the stress is constant.

❖ Hooke's law

This fundamental law of elasticity was proposed by Robert Hooke in 1679 and it states that "*Provided the strain is small, the stress is directly proportional to the strain*". In other words, *the ratio of stress to strain is a constant quantity for the given material* and it is called the modulus of elasticity or coefficient of elasticity.

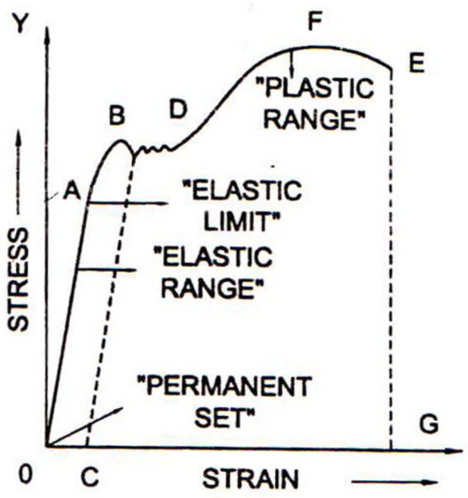
$$Stress\ \propto Strain$$
$$\therefore\ Stress = E \times Strain$$
$$\therefore E = \frac{Stress}{Strain}$$

The units and dimensions of the modulus of elasticity are the same as that of stress.

❖ Elastic Limit:

When the stress is continually increased in the case of solid, a point is reached at which the strain increased more rapidly. The stress at which the linear relationship between stress and strain hold good is called elastic limit of the material.

❖ Stress-Strain Diagram:



- The direct proportionality between stress and strain is found to be true only for small values of strain as shown in the figure.
- The portion OA of the curve is a straight line showing that the stress is directly proportional to strain. It shows that Hooke's law is strictly obeyed up to the value of stress corresponding to point A. This point is called ***Elastic limit***.
- Beyond point A, the curve is not a straight line. In this region AB, the strain increases more rapidly than the stress and the behavior is partly elastic and partly plastic. If the object is unloaded at B, it does not come back to its original condition along path AO, but takes the dotted path BC. The object is said to have acquired ***permanent set***. And OC is called the ***residual strain***.
- Beyond the point B, the length of the wire starts increasing without any increase in stress. Thus, wire begins to flow after point B and it continues up to D. The point B, at which the wire begins to flow is called ***yield point***.
- Beyond the point F, the graph indicates that length of the wire increases, even if the wire is unloaded. The wire breaks ultimately at point E, called the ***breaking point*** of the wire. The portion of the graph between D and E is called the ***plastic region***.

❖ Three types of elasticity:

There are three types of strain, therefore we have three types of elasticity.

(1) Linear elasticity called ***Young's modulus***, corresponding to *linear strain*.
(2) Elasticity of volume or ***Bulk modulus***, corresponding to *volume strain*.
(3) Elasticity of shape or shear modulus or ***Modulus of Rigidity***, corresponding to *shear strain*.

(1) Young's Modulus:

- When the deforming force is applied to the body only along a particular direction, the change per unit length in that direction is called *longitudinal, linear or elongation strain*.
- The force applied per unit area of cross section is called *longitudinal or linear stress*.
- The ratio of longitudinal stress to linear strain, within the elastic limit, is called Young's modulus.
- It is denoted by Y

$$Y = \frac{Longitudinal\ stress}{Linear\ strain}$$

- Consider a wire of length L having area of cross section 'a', fixed at one end and loaded at the other end.
- Suppose that a normal force F is applied to the free end of the wire and its length increase by l.

$$Longitudinal\ stress = \frac{F}{a} \quad and\ \ linear\ strain = \frac{l}{L}$$

$$Y = \frac{\frac{F}{a}}{\frac{l}{L}} \quad or \quad Y = \frac{FL}{al}$$

- Young's modulus can also be defined as the force applied to a wire of unit length and unit cross sectional area to produce the increase in length by unity.
- The units of Young's modulus are **Pascal or N/m²** in MKS and **dyne/cm²** in CGS system.

(2) Bulk Modulus:

- It is defined as the ratio of the normal stress to the volume strain**.**
- It is denoted by K. The bulk modulus is also known as the coefficient of cubical elasticity.

$$K = \frac{Normal\ stress}{Volume\ strain}$$

- Consider a cubic of volume V and surface area 'a'. Suppose that a force F which acts uniformly over the whole surface of the cubic, produces a decrease in its volume by v then,

$$Normal\ stress = \frac{F}{a} \quad and \quad volume\ strain\ = \frac{v}{V}$$

$$\therefore K = \frac{FV}{av}$$

Now, the pressure is $P = \frac{F}{a}$

$$\therefore K = \frac{-PV}{v}$$

- If the volume increase on increasing the stress the bulk modulus given by

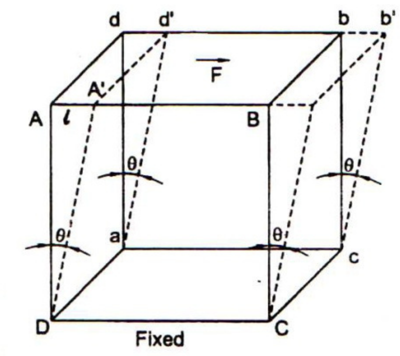
$$K = \frac{PV}{v}$$

- The units of bulk modulus are Pa or N/m² in SI.
- **Compressibility:** The reciprocal of the bulk modulus of a material is called compressibility i.e. **1/K.**

(3) Modulus of Rigidity:

- It is defined as the ratio of tangential stress to shear strain.
- It is also called shear modulus. It is denoted by η.

$$\eta = \frac{Tangential\ stress}{Shear\ strain}$$



- Consider a rectangular block, whose lower face **aDCc** is fixed and the upper face **ABbd** is subjected to tangential force F.
- Let 'a' be the area of the each face and AD = L be the perpendicular distance between them.
- The tangential force will displace the upper face of parallelepiped by a distance AA' = l.

- If $\angle ADA' = \theta$, then θ is the angle of shear.

$$Tangential\ stress = \frac{F}{a} \qquad and \quad Shear\ strain = Angle\ of\ shear = \theta$$

$$\eta = \frac{\frac{F}{a}}{\theta} = \frac{F}{a\theta}$$

- For solids, angle of shear is very small, so in ΔDAA'

$$\theta \approx tan\theta = \frac{AA'}{AD} = \frac{l}{L}$$

- The distance 'l' through which the upper face has been displaced is called lateral displacement.

$$\therefore \eta = \frac{F\,L}{a\,l}$$

❖ Work done per unit volume in case of elongation strain:

- Consider a wire of length l and area of cross section 'a' suspended from a rigid support.
- Suppose that a normal force 'F' is applied at its free end and its length increases by dl.
- The work done for a small displacement dl is given by

$$dW = F\ dl \qquad \ldots\ldots\ldots\ldots (1)$$

- We know that,

$$Y = \frac{\frac{F}{a}}{\frac{l}{L}}$$

$$\therefore F = \frac{Y\,a\,l}{L}$$

Substituting this value of F in above equation (1), we get

$$dW = \frac{Y\,a\,l}{L}\ dl$$

- Therefore, the total work done for the stretching a wire of length 'l' given by,

$$W = \int_0^l dW$$

$$W = \int_0^l \frac{Yal}{L}\ dl$$

$$W = \frac{Ya}{L}\int_0^l l\ \ dl$$

$$W = \frac{Y\,a}{L}\left(\frac{l^2}{2}\right)$$

$$W = \frac{1}{2}\frac{Y\,a\,l}{L} \times l$$

$$W = \frac{1}{2}\ F \times \mathrm{l}$$

$$\therefore Total\ work\ done\ W = \frac{1}{2}\ straching\ force \times change\ in\ length$$

- This work done stored in form of potential energy.

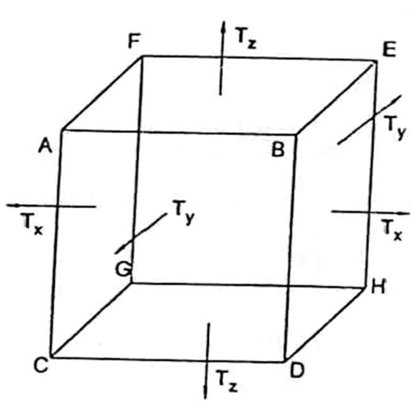
- Now, the volume of the wire = $a\,l$

$$\therefore Work\ done\ per\ unit\ volume = \frac{1}{2}\frac{F \times l}{a \times L}$$

$$= \frac{1}{2}\left(\frac{F}{a}\right) \times \left(\frac{l}{L}\right)$$

$$\therefore Work\ done\ per\ unit\ volume\ of\ the\ wire = \frac{1}{2} \times stress \times strain$$

❖ Deformation of cube –Bulk Modulus:



Let us consider a unit cube ABCDEFGHA. Suppose force T_x, T_y and T_z are acting perpendicular to the faces BEHD and AFGC, ABDC and EFGH, ABEF and DHGC respectively, as shown in figure.

Let 'α' be the increase per unit length per unit tension along the direction of the force and 'β' be the contraction produced per unit length per unit tension direction perpendicular to the force.

Due to the applied force, the elongations produce in the edges AB, BE and BD are $T_x\alpha$, $T_y\alpha$ *and* $T_z\alpha$ respectively. Similarly, the contraction produced in the perpendicular to these edges will be $T_x\beta$, $T_y\beta$ *and* $T_z\beta$.

The length of edges after elongation and contraction becomes,

$$AB = 1 + T_x\alpha - T_y\beta - T_z\beta$$

$$BE = 1 + T_y\alpha - T_x\beta - T_z\beta$$

$$BD = 1 + T_z\alpha - T_x\beta - T_y\beta$$

The volume of cube now becomes

$$V' = AB \times BE \times BD$$

$$V' = \left(1 + T_x\alpha - T_y\beta - T_z\beta\right) \times \left(1 + T_y\alpha - T_x\beta - T_z\beta\right) \times (1 + T_z\alpha - T_x\beta - T_y\beta)$$

$$V' = 1 + (\alpha - 2\beta)\,(T_x + T_y + T_z)$$

Neglecting squares and products of α and β.

In the case of bulk modulus, the force acting uniformly in all the directions,

Hence, $T_x = T_y = T_z$

$$\therefore V' = 1 + (\alpha - 2\beta)\,3T$$

The original volume of cube is unity; therefore increase in volume of the cube

$$V' = 1 + (\alpha - 2\beta)\,3T - 1$$

$$V' = (\alpha - 2\beta)\,3T$$

If pressure P is applied instead of tension T out words, the cube compressed and the volume decreased by the amount $3P(\alpha - 2\beta)$.

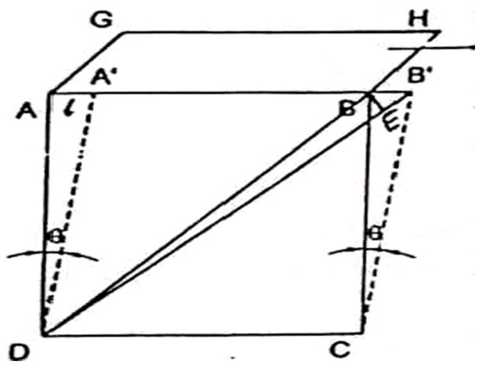
$$Volume\ strain\ = \frac{Change\ in\ volume}{Original\ volume} = \frac{3P(\alpha - 2\beta)}{1}$$

$$Bulk\ modulus\ K\ = \frac{Stress}{Volume\ strain} = \frac{P}{3P(\alpha - 2\beta)}$$

$$\therefore K\ = \frac{1}{3(\alpha - 2\beta)}$$

$$Compressibility = \frac{1}{K} = \frac{1}{3(\propto - 2\beta)}$$

❖ Modulus of rigidity:



Consider a cube with an edge 'L'. Let shearing force $\vec{F}$ be applied on the tope face ABHG of a cube, which produce shear by an angle θ and linear displacement 'l'.
The face ABCD becomes A'B'CD.

$$Tensile\ stess = \frac{F}{area\ of\ face\ ABHG}$$

$$= \frac{F}{L^2} \quad = T$$

$$Shear\ strain\ = \frac{l}{L}$$

$$\therefore Modulus\ of\ rigidity\ \eta = \frac{Tensile\ stess}{Shear\ strain} = \frac{T}{\theta}$$

A shearing stress along AB is equivalent to a tensile stress along DB and an equal compression stress along CA at right angles.

If α and β are the longitudinal and lateral strains per unit stress respectively.

Then extension along diagonal DB due to tensile stress = $DB\ T\ \alpha$ and, extension along diagonal DB due to compression stress along AC = $DB\ T\ \beta$.

Therefore, the total extension along DB = $DB\ T(\ \alpha + \beta)$

But, from above figure diagonal $DB\ = \sqrt{L^2 \times L^2}$

$$\therefore DB\ = \sqrt{2}L$$

Therefore, the total extension of diagonal $EB'\ = \sqrt{2}\ L\ T(\ \alpha + \beta)$ ... ... (1)

$$\text{In } \Delta\ BB'E,\ Cos\ BB'E = \frac{EB'}{BB'}$$

$$\therefore EB'\ = BB'\ Cos\ \angle BB'E$$

$$But, BB' = l\ and\ \angle BB'E\ = 45^0$$

$$\therefore EB' = l\ Cos\ 45^0$$

$$\therefore EB'\ = \frac{l}{\sqrt{2}} \qquad \text{... ... ... (2)}$$

Now, comparing equation(1) and (2), we get

$$\therefore \frac{l}{\sqrt{2}} = \sqrt{2}\ L\ T(\ \alpha + \beta)$$

$$\therefore \frac{L\ T}{l}\ = \frac{1}{2(\ \alpha +\ \beta)}$$

$$\therefore \frac{T}{\frac{l}{L}}\ = \frac{1}{2(\ \alpha +\ \beta)}$$

$$\therefore \frac{T}{\theta}\ = \frac{1}{2(\ \alpha +\ \beta)}$$

$$\therefore \eta\ = \frac{1}{2(\ \alpha +\ \beta)}$$

❖ Young's Modulus:

Let us consider unit tension applied on the edge of the unit cube, which produces the extension $'\alpha'$ linear stress =1 and linear strain = $\frac{\propto}{1}$ = α.

$$Young's\ modulus = Y = \frac{1}{\alpha}$$

❖ Relation connecting the Elastic Constants:

We know that

$$K = \frac{1}{3(\alpha - 2\beta)}$$

and

$$\eta = \frac{1}{2(\alpha + \beta)}$$

$$(\alpha - 2\beta) = \frac{1}{3K} \qquad \dots\dots\dots\dots (1)$$

$$(\alpha + \beta) = \frac{1}{2\eta} \qquad \dots\dots\dots\dots (2)$$

Subtracting (1) and (2)

$$3\beta = \frac{1}{2\eta} - \frac{1}{3K}$$

$$3\beta = \frac{3K - 2\eta}{6\eta K}$$

$$\beta = \frac{3K-2\eta}{18\eta K} \qquad \dots\dots\dots\dots (3)$$

Multiplying equation(2) by 2 and adding equations (1) and (2) we get,

$$3\alpha = \frac{1}{\eta} + \frac{1}{3K}$$

$$3\alpha = \frac{3K + \eta}{3K\eta}$$

$$\alpha = \frac{3K+\eta}{9K\eta} \qquad \dots\dots\dots\dots (4)$$

Form equation of young's modulus,

$$Y = \frac{1}{\alpha} \quad i.e\ \alpha = \frac{1}{Y} \qquad \dots\dots\dots\dots (5)$$

Using equation (5) in (4)

$$\frac{1}{Y} = \frac{3K + \eta}{9K\eta}$$

$$\therefore \frac{9}{Y} = \frac{3K}{K\eta} + \frac{\eta}{K\eta}$$

$$\therefore \frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K} \qquad \dots\dots\dots\dots (6)$$

The above equation gives the relation connecting the three elastic constants *Y, K* and η .

❖ Poisson's Ratio:

When we stretch a wire, it becomes longer but thinner. The increase in its length is always accompanied with decrease in its cross section.

The strain produced along the direction of the applied force is called *primary or linear or tangential strain* (α) and strain produced at right angle to the applied force is called *secondary or lateral strain* (β).

Within the elastic limit, the lateral strain (β) is proportional to the linear strain (α) and the ratio between them is a constant, called Poisson's ratio (σ).

$$\sigma = \frac{Lateral\ strain}{linear\ strain} = \frac{\beta}{\alpha}$$

If the body under tension suffers no lateral strain then Poisson's ratio is zero.

❖ Limiting values of 'σ' :

We know that,

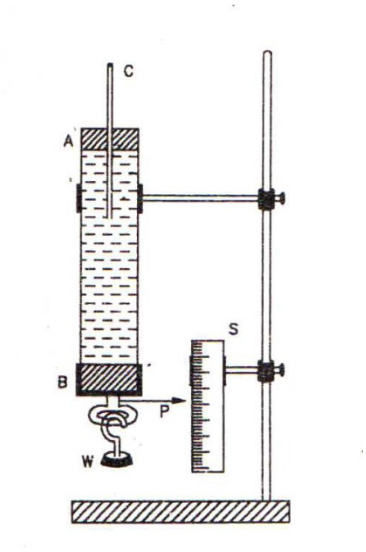
$$3K(1-2\sigma) = 2\eta\,(1+\sigma)$$

Where, K and η are essentially positive quantities.

- Now if σ is positive, then the RHS and hence LHS must be positive.
  This is true, if $1-2\sigma > 0$
  $$\therefore 2\sigma < 1$$
  $$\therefore \quad \sigma < \frac{1}{2}$$
  $$\therefore \sigma < 0.5 \qquad \ldots\ldots\ldots\ldots (1)$$
- If σ is negative, then the LHS and hence RHS must be positive.
  This is true, if $1+\sigma > 0$
  $$\therefore \quad \sigma > -1$$
  $$\therefore \ -1 < \sigma \qquad \ldots\ldots\ldots\ldots (2)$$
  Combining relation (1) and (2), we have
  $$-1 < \sigma < 0.5 \qquad \ldots\ldots\ldots\ldots (3)$$

Thus the limiting values of σ are -1 and 0.5. In actual practice, the value of σ lie between 0.2 to 0.4.

❖ Determination of Poisson's Ratio for Rubber:



To determine the value of σ for rubber, we take about a meter long tube AB and suspended vertically as shown in figure. Its two ends are property stopper with rubber corks and liquid glue. A glass tube C of half meter long and 1 cm in diameter is fitted vertically into the cork A through a suitable hole. A suitable weight W is then suspended from the lower end of the tube. This will increase the length and the internal volume of the tube.

It results in the fall of the level of meniscus in glass tube C. Both the increase in length (dL) and the decrease in the meniscus level (dh) are measured.

Let L, D and V be the original length, diameter and volume of the tube respectively. Then the area of cross-section of tube is

$$A = \pi r^2 = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{2} \qquad \ldots\ldots\ldots\ldots (1)$$

Differentiating above equation, We have

$$dA = \frac{\pi}{4} 2D\,dD = \frac{\pi D}{2} dD$$

$$\therefore \ dA = \frac{\pi D}{2} dD \ \ \frac{D}{2} \ \ \frac{2}{D}$$

$$\therefore dA = \frac{\pi D^2}{4} dD \ \ \frac{2}{D}$$

$$\therefore \ dA = \frac{2A\,dD}{D} \qquad \ldots\ldots\ldots\ldots (2)$$

Now, the increase in length of rubber tube dL and the increase in volume dV are accompanied with the decrease in area of cross section dA.

$$\text{Volume} = \text{area of cross section} \times \text{length}$$

$$V + dV = (A - dA)(L + dL)$$

$$\therefore \ V + dV = A\,L + A\,dL - dA\,L - dA \cdot dL \qquad \ldots\ldots\ldots\ldots (3)$$

Neglecting $dA \cdot dL$ being very small.

We have,

$$V + dV = A\,L + A\,dL - dA\,L$$
$$\therefore\ V + dV = V + A\,dL - dA\,L$$
$$\therefore\ dV = A\,dL - dA\,L \qquad \ldots\ldots\ldots\ldots(4)$$

Substituting the value of dA, we get,

$$dV = A\,dL - \frac{2A\,dD}{D}\,L \qquad \ldots\ldots\ldots\ldots(5)$$

Dividing by dL on both sides,

$$\frac{dV}{dL} = A - \frac{2AL}{D}\,\frac{dD}{dL}$$

$$\therefore\ \frac{2AL}{D}\,\frac{dD}{dL} = A - \frac{dV}{dL}$$

$$\therefore\ \frac{dD}{dL} = \frac{D}{2AL}\left[A - \frac{dV}{dL}\right]$$

$$\therefore\ \frac{dD}{dL} = \frac{D}{2L}\left[\frac{A}{A} - \frac{1}{A}\frac{dV}{dL}\right]$$

$$\therefore\ \frac{dD}{dL} = \frac{D}{2L}\left[1 - \frac{1}{A}\frac{dV}{dL}\right] \qquad \ldots\ldots\ldots\ldots(6)$$

Now Poisson's ratio is given by

$$\sigma = \frac{Lateral\ Strain}{Liner\ Strain} = \frac{dD/D}{dL/L}$$
$$\therefore \sigma = \frac{L}{D} \times \frac{dD}{dL} \qquad \ldots\ldots\ldots\ldots(7)$$

Substituting the value of dD/dL from equation (6) in (7), we get

$$\sigma = \frac{L}{D} \times \frac{D}{2L}\left[1 - \frac{1}{A}\frac{dV}{dL}\right]$$
$$\therefore \sigma = \frac{1}{2}\left[1 - \frac{1}{A}\frac{dV}{dL}\right] \qquad \ldots\ldots\ldots\ldots(8)$$

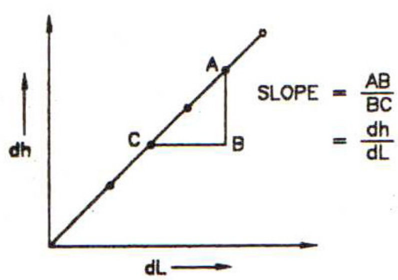
If r be the internal radius of the tube, so that $A = \pi r^2$

$$\therefore \sigma = \frac{1}{2}\left[1 - \frac{1}{\pi r^2}\frac{dV}{dL}\right] \qquad \ldots\ldots\ldots\ldots(9)$$

If 'a' be the internal radius of the capillary tube, we have $dV = \pi a^2 \cdot dh$

$$\therefore \sigma = \frac{1}{2}\left[1 - \frac{1}{\pi r^2}\frac{\pi a^2 \cdot dh}{dL}\right]$$

$$\therefore \sigma = \frac{1}{2}\left[1 - \frac{a^2}{r^2}\frac{dh}{dL}\right] \qquad \ldots\ldots\ldots\ldots(10)$$



The value of 'a' and 'r' are determined by a travelling microscope and a vernier caliper respectively and the average value of $dh/_{dL}$ is obtained from the slope of the straight line graph by plotting a number of corresponding value of dh against the dL as shown in fig.

Solved Numerical

Ex-1 The Young's modulus of a metal is 2 X 10^{11} N/m^2 and its breaking stress is 1.078 X 10^9 N/m^2. Calculate the maximum amount of energy per unit volume which can be stored in the metal when stretched.

Sol: Here, Y = 2 X 10^{11} N/m^2

Maximum stress = 1.078 X 10^9 N/m^2

$$\text{Energy stored per unit volume } = \frac{1}{2}\text{ stress } \times \text{strain}$$

$$= \frac{1}{2}[1.078 \times 10^9] \times \frac{\text{stress}}{\text{Y}}$$

$$= \frac{1}{2} \times 1.078 \times 10^9 \times \frac{1.078 \times 10^9}{2 \times 10^{11}}$$

$$= 2.90 \times 10^6 \text{ J/m}^2$$

Ex-2 Find the work done in stretching a wire of 1 sq. mm cross section and 2 m long through 0.1 mm. Given Y = 2 X 10^{11} N/m^2.

Sol: As we know

$$Work\ done\ in\ stretching\ a\ wire = \frac{1}{2} \times Stretching\ force \times stretch$$
$$= \frac{1}{2} \times F \times l$$
$$= \frac{1}{2} \times \frac{Yal}{L} \times l$$

Here, Y = 2 X 10^{11} N/m^2

L = 2 m.

l = 0.1 mm = 10^{-4} m

a = 1 sq.mm = 10^{-6} m^2

$$\therefore Work\ done\ = \frac{1}{2} \times \frac{2 \times 10^{11} \times 10^{-6} \times 10^{-6} \times 10^{-4}}{2} = 5 \times 10^{-4}\ joules.$$

Ex: 3 The modulus of rigidity and Poisson's ratio of the material of a wire are 2.87 x 10^{10} N/m^2 and 0.379 respectively. Find the value of Young's modulus of the material of the wire.

Sol: Here, η = 2.87 x 10^{10} N/m^2 and σ = 0.379

We know that,

$$\eta = \frac{Y}{2\,(1 + \sigma)}$$

$$\therefore Y = 2\eta\,(1 + \sigma)$$

$$\therefore\ Y = 2 \times 2.87 \times 10^{10}(1 + 0.379)$$

$\therefore Y = 7.915 \times 10^{10}\ N/m^2$

Ex: 4 A steel wire, 1 meter long and 1 mm square in cross section, supports a mass of 6 kg. By how much does it stretch? (Give $Y = 20 \times 10^{10}\ N/m^{-2}$)

Sol: Here, $L = 1$ m

$A = 1$ sq. mm $= 10^{-6}$ sq. m

$m = 6$ kg.

$l = ?$

The stretching force $F = mg = 6 \times 9.8 = 58.8$ N

Young modulus $Y = \frac{F/a}{l/L}$

$$\therefore l = \frac{FL}{ay}$$

$$\therefore l = \frac{58.8 \ \times 1}{10^{-6} \times 20 \times 10^{10}}$$

$$\therefore l = 2.94 \ \times 10^{-4}\ m$$

$\therefore$ The increase in length of wire = 0.294 m

Ex: 5 A bronze bar 1.7 m long and 50 mm in diameter is subjected to a tensile stress of 70 Mega Newton / m^2. Calculate the extension produced in the bar and work done during the process. The value of Young's modulus for the material of the bar may be taken to be 85×10^9 N / m^2.

Sol:

Since Y = stress/strain, we have strain = stress / Y

And

Extension produced = strain x length

$$= \frac{stress}{Y} \times length = \frac{70 \times 10^6}{85 \times 10^9} \times 1.7 = 14 \ \times \ 10^{-4}\ m = 1.4\ mm$$

Now work done during stretch = 0.5 x stretching force x stretch

Here stretching force = tensile stress x area of cross section of the rod

$$= (\ 70 \ \times 10^6) \ \times \pi \times 50^2 \times 10^{-6} \ / \ 4$$

$$\therefore work\ done = \frac{1}{2} \times \frac{(\ 70 \ \times 10^6) \ \times \pi \times 50^2 \times 10^{-6}}{4} \times 14 \times 10^{-4}$$

$$= \frac{7 \ \times \pi \times 35}{8} = 96.23\ joules$$

Exercise

(1) A wire 300 cm long and .625 sq. cm in cross – section is found to stretch 0.3 cm under a tension of 1200 kilograms. What is the Young's modulus of the material of the wire?

(Ans. : 2.3×10^{12} dynes / sq.cm.)

(2) Calculate the work done in stretching a uniform metal wire of area of cross section 10^{-6} m^2 and length 1.5 m through 4×10^{-3} m. Given $Y = 2 \times 10^{11}$ N / m^2.

(Ans. : 1.066 Joule)

(3) Calculate the Poisson's ratio for the material given, $Y = 12.25 \times 10^{10}$ N / m^2 and $\eta = 4.55 \times 10^{10}$ N / m^2. (Ans.: 0.346)

Question Bank

Multiple Choice Questions:

(1) Hooke's law essentially defines ________

(a) Stress (b) Strain

(c) Yield point (d) Elastic limit

(2) The dimensional formula of stress is _______

(a) $[M^0L^1T^2]$ (b) $[M^1L^{-1}T^{-2}]$

(c) $[M^1L^{-1}T^{-2}]$ (d) $[M^1L^{-1}T^{-1}]$

(3) The nearest approach to the perfectly elastic body is ________

(a) Quarts fibre (b) Putty

(c) Silver (d) Platinum

(4) _______ is the perfectly plastic material

(a) Quarts fibre (b) Putty

(c) Silver (d) Platinum

(5) The restoring force per unit area is called _________

(a) Stress (b) Strain

(c) Elasticity (d) Plasticity

(6) The change per unit dimension of the body is called ________

(a) Stress (b) Strain

(c) Elasticity (d) Plasticity

(7) The restoring force per unit area perpendicular to the surface is called _________

(a) Longitudinal Stress (b) Tangential Stress

(c) Normal Stress (d) Tensile Stress

(8) The restoring force per unit area perpendicular to the surface is called _________

(a) Longitudinal Stress (b) Lateral Stress

(c) Normal Stress (d) Tensile Stress

(9) The strain produced at right angles to the direction of force is called ________

(a) Primary strain (b) Secondary strain

(c) Volume strain (d) Shear strain

(10) Compressibility of a material is reciprocal of ____________

(a) Modulus of rigidity (b) Young Modulus

(c) Bulk Modulus (d) Coefficient of rigidity

(11) The work done per unit volume in stretching the wire is equal to ________

(a) Stress x strain (b) (1/2) stress x strain

(c) Stress / strain (d) Strain / stress

(12) When a body undergoes a linear tensile strain it experiences a lateral contradiction also. The ratio of lateral strain to longitudinal strain is known as:

(a) Young's modulus (b) Bulk modulus

(c) Poisson's ratio (d) Hooke's Law

(13) Theoretical value of Poisson's ratio lies between _________

(a) -1 and + 0.5 (b) - 1 and -2

(c) -0.5 and +1 (d) -1 and 0

(14) Which of the following relations is true?

(a) $3\alpha = \frac{3K - \eta}{3K\eta}$ (b) $3\alpha = \frac{3K + \eta}{3K\eta}$

(c) $\alpha = \frac{3K + \eta}{3K\eta}$ (d) $\alpha = \frac{3K - \eta}{3K\eta}$

(15) The relationship between Y, η and σ is ___________

(a) $Y = 2\eta\,(1 + \sigma)$ (b) $\eta = 2Y\,(1 + \sigma)$

(c) $\sigma = 2Y / (1 + \eta)$ (d) $Y = \eta\,(1 + \sigma)$

(16) Which of the relation is true?

(a) $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$ (b) $\sigma = \frac{3K + 2\eta}{6K - 2\eta}$

(c) $\sigma = \frac{6K + 2\eta}{6K - 2\eta}$ (d) $\sigma = \frac{2K + 3\eta}{2K - 6\eta}$

(17) The Poisson's ratio cannot have the value ________

(a) 0.7 (b) 0.2

(c) 0.1 (d) -0.52

(18) The Poisson's ratio can have the value _________

(a) 0.7 (b) -1.1

(c) 1.0 (d) 0.49

(19) Units of modulus of elasticity is ________

(a) dyne/cm (b) dyne/cm^2

(c) N/m (d) Dyne

(20) The relation between K, α and β is _________

(a) $K = \frac{1}{2(\alpha - 2\beta)}$ (b) $K = \frac{1}{3(\alpha - 2\beta)}$

(c) $K = \frac{1}{(\alpha - 2\beta)}$ (d) $K = \frac{1}{3(\alpha + 2\beta)}$

(21) In Bulk modulus, there is a change in the volume of the body but no change in ____

(a) Size (b) Shape

(c) Line (d) Angle

(22) Increase in the length of a wire is always accompanied by a decrease in ________

(a) Length (b) Breadth

(c) Cross section (d) Height

(23) The ratio of Longitudinal stress to linear strain is called ________

(a) Modulus of rigidity (b) Young Modulus

(c) Bulk Modulus (d) Coefficient of rigidity

(24) The ratio of Tensile stress to shear strain is called ________

(a) Modulus of rigidity (b) Young Modulus

(c) Bulk Modulus (d) Poisson's ratio

Short Questions:

(1) Explain (i) elasticity and (ii) plasticity.

(2) Explain: (i) load (ii) stress and (iii) strain.

(3) Explain: (i) Normal stress and (ii) Tangential stress.

(4) Explain: (i) linear strain (ii) volume strain and (iii) Shear strain.

(5) Define Young's modulus and Bulk Modulus.

(6) Define Modulus of rigidity

(7) Explain: Young Modulus and Modulus of rigidity

(8) State and explain Hook's law.

(9) Define and explain Poisson's ratio.

(10) Show that the value of Poisson's ratio lies between -1 and + 0.5.

(11) Describe stress-strain diagram for elongation of a wire.

(12) Derive relation between Young's modulus (Y), Modulus of rigidity (η) and Poisson's ratio (σ)

Prove the relation, (i) $\frac{Y}{2\eta} = 1 + \sigma$ and (ii) $\frac{Y}{3K} = 1 - 2\sigma$.

Long Questions:

(1) Derive the formula for the work done per unit volume in stretching a wire.

(2) Prove that the work done per unit volume in stretching the wire is equal to $\frac{1}{2}$(stress × strain).

(3) Explain three elastic constants in detail.

(4) Show the bulk modulus of elasticity is $K = \frac{1}{3(\alpha - 2\beta)}$

(5) Derive the expression of bulk modulus for deformation of a cube

(6) Derive the relation $\eta = \frac{1}{2(\alpha+\beta)}$ for deformation of cube

(7) Derive the expression of modulus of rigidity for deformation of a cube

(8) Discuss the case of deformation of a cube and derive the necessary expression for Three elastic constants and hence prove that $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$

(9) Define Poisson's ratio and describe a method of determination. Derive the formula used.

(10) Define Poisson's ratio (σ) and derive formula of it to determine experimentally, i.

e. $\sigma = \frac{1}{2}\left(1 - \frac{1}{A} \quad \frac{dV}{dL}\right)$

(11) Derive the relation between three types of elastic module Y, K and η

(12) Define Poisson's ratio. Describe an experiment with necessary theory to determine the Poisson's ratio for rubber.

Answer key of MCQ:

(1)	(d)	(2)	(c)	(3)	(a)	(4)	(b)
(5)	(a)	(6)	(b)	(7)	(c)	(8)	(d)
(9)	(b)	(10)	(c)	(11)	(b)	(12)	(c)
(13)	(a)	(14)	(b)	(15)	(a)	(16)	(a)
(17)	(a)	(18)	(d)	(19)	(b)	(20)	(b)
(21)	(b)	(22)	(c)	(23)	(b)	(24)	(a)